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Quaternion Observer-Based Model-Independent Attitude Tracking Control of Spacecraft

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Based on the separation property for a rigid-body tracking control problem established by Seo and Akella, this work explores a model-independent and observer-based attitude tracking control method in which not only the unavailability of direct/accurate measurement of quaternion attitude is considered, but the inevitable factors of external disturbances and uncertain (or even time-varying) system parameters as well as actuation saturation are also explicitly addressed. The proposed control scheme is essentially model-independent in that the system dynamic model (valid for any rigid-body spacecraft) is used for stability analysis only, but not actually needed for setting up and implementing the proposed control strategy. As such, there is no need for extensively redesigning or reprogramming the control algorithms, even if unexpected external disturbances occur or system parameters/dynamics change during system operation. Those features, highly desirable in practice, are conformed and verified via theoretical analysis and numerical simulations.

I. Introduction

As ATTITUDE adjustment is one of the fundamental maneuvers that any spacecraft needs to frequently perform during its operation, extensive studies on attitude control systems for rigid-body spacecraft have been carried out during the past decades (e.g., [1–15], just to name a few). By assuming the availability of direct and exact measurement of both the body angular rates and the globally valid (nonsingular) quaternion vector, various full-state feedback attitude control schemes have been developed.

In practice, however, there exists no physical sensor that permits precise and direct measurement of the quaternion vector or any other representation of the body attitude; the attitude quaternion is always obtained indirectly from accompanying filters/observers that are driven by measurements from rate gyros, sun sensors, star sensors, earth sensors, magnetometers, and a host of other sensor candidates [16]. As such, it is highly desirable to develop partial-state feedback attitude control schemes that do not rely on direct measurement of attitude (quaternion) variables. Several interesting results in constructing attitude estimators (observers) and integrating them into partial-state feedback attitude control designs have been reported [16–18]. In terms of theory, Seo and Akella's work [16] has explicitly addressed the closed-loop stability issue when the attitude estimates generated by observers instead of the actual (true) attitude variables are adopted in the feedback control schemes, and established a separation property for the rigid-body attitude tracking control problem that delivers global stability for the composite closed-loop system.

It is important to note that, when it comes to implementation, not only the unavailability of direct measurement of attitude variables needs to be considered, other inevitable factors such as external disturbances, modeling uncertainties, as well as actuation constraints should also be addressed simultaneously. In this work, a new attitude control scheme that does not rely on direct exact measurement of attitude (quaternion) variables is developed. In contrast to most existing quaternion observer-based control, the proposed method does not need precise information of inertia matrix or any other system parameters. Also, the potential effect of external disturbances on system performance is explicitly addressed in the control design. The proposed control scheme is essentially model-independent in that the dynamic model is used for stability analysis only, but never actually needed in setting up and implementing the proposed control scheme. Furthermore, as actuation limit always exists in practice, the developed control algorithms are derived with explicit consideration of torque limits. The results reported in this paper are motivated in part by the early work of Cai et al. [15]. They are also closely related to the work of Seo and Akella [16]. In fact, the results given in this paper can be viewed as a natural extension of the work in [15,16].

The remainder of the paper is organized as follows: Section II reviews the basic concepts and relations on attitude representation. In Sec. III, robust adaptive attitude control algorithms independent of system parameters/dynamic model and direct/exact measurement of quaternion vector are presented. The issue of actuation constraints is addressed in Sec. IV. Simulation results are presented in Sec. V, and the paper is closed in Sec. VI with comments.

II. Formulization of Attitude Tracking

A. Basics of Attitude Representation

It is well known that when the quaternion vector is used for attitude representation, the attitude tracking error $q_e = [q_{e0}, q_{ev}^T]^T$ can be expressed in terms of the quaternion $q_b = [q_{b0}, q_{bv}^T]^T$ (of the body frame) and the quaternion $q_r = [q_{r0}, q_{rv}^T]^T$ (of reference frame) as follows [1]:

$$q_e = q_b \otimes q_r^{-1} = \begin{bmatrix} q_{r0}q_{b0} + q_{rv}^T q_{bv} \\ q_{r0}q_{bv} - q_{b0}q_{rv} + S(q_{bv})q_{rv} \end{bmatrix} \tag{1}$$

where $S(.)$ is a skew-symmetry matrix operator defined by $S(q_{bv})q_{rv} = q_{bv} \times q_{rv}$, where hereafter "×" represents the cross-product operator. Furthermore,

$$\dot{q}_e = \tfrac{1}{2}E(q_e)\omega_e \tag{2}$$

and

$$\omega_e = \omega_b - R(q_e)\omega_r \tag{3}$$

where ω_e is the angular velocity tracking error,

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$$E(q_e) = [-q_{ev}, q_{e0}I_{3\times3} - S(q_{ev})]^T$$

stands for the quaternion operator, and

$$R(q_e) = (q_{e0}^2 - q_{ev}^T q_{ev})I_{3\times3} + 2q_{ev}q_{ev}^T - 2q_{e0}S(q_{ev})$$

represents the rotational matrix.

B. Attitude Observer

Most attitude tracking control schemes are developed based on the assumption that q_b is available [1–15]. However, in practice, the attitude quaternion cannot be physically measured directly by attitude sensors. Thus, an attitude observer is usually needed to estimate the attitude quaternion values for feedback control. Following the work in [16,19], we consider the following quaternion measurement model and its estimator:

$$y(t) = R[q_b(t)]x(t) \tag{4}$$

$$\hat{y}(t) = R[q_o(t)]x(t) \tag{5}$$

where $y(t)$ denotes the actual attitude measurement signal from attitude measurement sensor, $\hat{y}(t)$ is the estimated measurement in the observation frame $\mathcal{O}$, $x(t)$ is the measurement reference signal (unit vector governing the inertial direction of observation), and $\dot{x}(t)$ is assumed to be bounded with time [16]; $q_o(t)$ represents the estimated value of $q_b(t)$ in observation frame $\mathcal{O}$, which is updated by

$$\dot{q}_o(t) = \tfrac{1}{2}E(q_o)(\omega_b + \gamma y \times \hat{y}), \qquad \gamma > 0 \tag{6}$$

The observation error of the observer, denoted by $\tilde{q}_e = [\tilde{q}_{e0}, \tilde{q}_{ev}^T]^T$ (difference between q_o and q_b), shares the following relation via the rotational matrix

$$R(\tilde{q}_e) = R^T(q_b)R(q_o) \tag{7}$$

and the estimated attitude tracking error, denoted by $\hat{q}_e = [\hat{q}_{e0}, \hat{q}_{ev}^T]^T$ (difference between q_o and q_r), bears the relation of

$$\hat{q}_e = q_o \otimes q_r^{-1} \tag{8}$$

We define estimated angular velocity tracking error of ω_e as

$$\hat{\omega}_e = \omega_b - R(\hat{q}_e)\omega_r \tag{9}$$

Then, it follows that

$$\dot{\hat{q}}_e = \tfrac{1}{2}E(\hat{q}_e)[\omega_b - R(\hat{q}_e)\omega_r + \gamma y \times \hat{y}] \tag{10}$$

Obviously, if $[\tilde{q}_{ev}, \hat{q}_{ev}, \hat{\omega}_e] \to 0$, then $q_o \to q_b$, $q_r \to q_o$, and $\omega_b \to \omega_r$ from Eqs. (7–9), such that the objective of attitude tracking control with a quaternion observer can be addressed by regulating

$$\lim_{t\to\infty}[\tilde{q}_{ev,}\hat{q}_{ev}, \hat{\omega}_e] = 0$$

Note that, from Eqs. (4–7), it is derived that [16]

$$\dot{\tilde{q}}_e = \begin{bmatrix} \dot{\tilde{q}}_{e0} \\ \dot{\tilde{q}}_{ev} \end{bmatrix} = \begin{bmatrix} \gamma\tilde{q}_{e0}\|\tilde{q}_{ev} \times x\|^2 \\ \gamma[\tilde{q}_{e0}^2(\tilde{q}_{ev} \times x) \times x - (x^T\tilde{q}_{ev})S(\tilde{q}_{ev})(\tilde{q}_{ev} \times x)] \end{bmatrix} \tag{11}$$

$$y \times \hat{y} = R(q_b)[2\tilde{q}_{e0}(\tilde{q}_{ev} \times x) \times x - 2(x^T\tilde{q}_{ev})(\tilde{q}_{ev} \times x)] \tag{12}$$

which will be applied to future control development.

The following lemma confirms the convergence of the observer defined in Eqs. (4–6).

Lemma 1 [16]: The observer defined in Eqs. (4–6) is asymptotic stable in that the estimated attitude quaternion q_o asymptotically approaches the actual attitude quaternion q_b as $q_b^T(T)q_o(T) \neq 0$ for some $T \geq 0$.

Remark 1: Fundamentally, Lemma 1 (see [16] for proof) ensures that the observer given by Eqs. (4–6) leads to

$$\lim_{t\to\infty}\tilde{q}_{ev} = 0$$

in practice, so that $q_o(t) \to q_b(t)$ as $t \to \infty$. This separation property established for a rigid-body spacecraft tracking control problem by Seo and Akella [16] will be used later on for our control design. Also note that $\tilde{q}_{e0}(t) = q_b^T(t)q_o(t)$ for all $t \geq 0$ from Eq. (7) [20], so that $q_b^T(0)q_o(0) = 0$ corresponds to the case of $\tilde{q}_{e0}(t) \equiv 0$ for $\forall t \geq 0$ from Eq. (11), which is an unstable equilibrium. Therefore, any imperceptible computation errors or any other disturbances would always make the condition $\tilde{q}_{e0}(T) = q_b^T(T)q_o(T) \neq 0$ valid practically for some $T > 0$ [16].

III. Model-Independent Attitude Control Design

Our objective is to develop a model-independent attitude tracking control scheme that does not explicitly rely on any information about the system model/parameter yet is robust against external disturbances. To this end, we introduce $\tau_d \in \mathbb{R}^3$ (external disturbance torque) as well as uncertain or even time-varying (due to, for instance, fuel burning or payload release/addition) system parameters (i.e., inertia matrix J) into the vehicle dynamic model to get

$$\frac{\mathrm{d}}{\mathrm{d}t}(J\omega_b) = -S(\omega_b)J\omega_b + \tau + \tau_d \tag{13}$$

where $J \in \mathbb{R}^{3\times3}$ is symmetric and positive-definite inertial matrix, and $\tau \in \mathbb{R}^3$ is the torque acting on the vehicle. Define the filtered estimate error variable

$$\hat{s}(\hat{\omega}_e, \hat{q}_{ev}) = \hat{\omega}_e + \beta\hat{q}_{ev}, \qquad \beta > 0 \tag{14}$$

Then, Eq. (13) can be expressed in terms of $\hat{s}$ as follows:

$$J\dot{\hat{s}} = \tau - \tfrac{1}{2}\dot{J}\,\hat{s} + L_{\hat{s}\tau_d}(.) \tag{15}$$

where

$$\begin{aligned} L_{\hat{s}\tau_d}(.) = &-S(\omega_b)J\omega_b + \beta J\dot{\hat{q}}_{ev} + \tfrac{1}{2}\dot{J}\,\hat{s} - \dot{J}\omega_b + \tau_d \\ &+ J[S(\hat{\omega}_e + \gamma y \times \hat{y})R(\hat{q}_e)\omega_r - R(\hat{q}_e)\dot{\omega}_r] \end{aligned} \tag{16}$$

The reason for adding $\frac{1}{2}\dot{J}\,\hat{s}$ to Eq. (16) and subtracting it from Eq. (15) is to facilitate stability analysis, as will be apparent later. Note that the lumped term $L_{\hat{s}\tau_d}(.)$ containing three parts: 1) system nonlinearities (depending on desired attitude trajectory and physical parameters, the inertia matrix in particular), 2) external disturbances (changing with operation conditions), and 3) the effect due to time-varying moment inertia matrix. One of the major challenges for attitude tracking control design stems from such an uncertain term in the dynamic equation. Obviously, a feasible and practical control scheme should not be based upon such $L_{\hat{s}\tau_d}(.)$ directly.

In this work, we explore a method that is based on the *core* information of $L_{\hat{s}\tau_d}(.)$ extracted from the following assumptions:

1) The inertial matrix $J(.)$ is symmetric positive definite, the entire operation of the vehicle, that is, $J^T = J > 0$.

2) Both $J(.)$ and $\dot{J}(.)$ are bounded for some unknown constants, that is, $\|J(.)\| \leq c_J$ and $\|\dot{J}(.)\| \leq c_{dJ}$ for unknown constants $c_J \geq 0$ and $c_{dJ} \geq 0$.

3) Both the reference angular velocity and its variation are bounded, that is, $\|\omega_r\| \leq c_w$ and $\|\dot{\omega}_r\| \leq c_{dw}$ for some unknown constants $c_w \geq 0$ and $c_{dw} \geq 0$.

4) The external disturbance is modeled by the boundary constraint $\|\tau_d\| \leq c_g + c_d\|\omega_b\|^2$ for $c_g \geq 0$ and $c_d \geq 0$ unknown but constants.

Note that the symmetric and positive-definite property of the inertial matrix is always true with rigid spacecraft, thus assumption 1) is reasonable. Assumption 2) is acceptable as long as the mass (including the fuel) of the spacecraft does not change infinitely fast during system operation. Assumption 3) is also necessary for practical and feasible attitude tracking. Also note that the external disturbance is related to gravitation, solar radiation, magnetic forces (all could be assumed bounded), and aerodynamic drags (propotional to the square of angular velocity). Therefore, for all those disturbances considered, assumption 4) is practical.

Based upon the preceding assumptions, the following inequalities are easily established:

$$\begin{aligned} &\|S(\omega_b)J\omega_b\| \le \|S(\omega_b)\|\|J\|\|\omega_b\| \le c_J\|\omega_b\|^2 \\ &\|\beta J\dot{\hat{q}}_{ev}\| \le \beta c_J\|\omega_b\| + \beta c_J(c_w+\gamma) \\ &\|0.5\dot{J}\,\hat{s} - \dot{J}\omega_b\| \le 0.5c_{dJ}(\|\hat{\omega}_e\|+\beta) \\ &\quad + c_{dJ}\|\omega_b\| \le c_{dJ}[0.5(\beta+c_w)+1.5\|\omega_b\|] \\ &\|JS(\hat{\omega}_e+\gamma y\times\hat{y})R(\hat{q}_e)\omega_r\| \le c_Jc_w(\|\omega_b\|+c_w+\gamma) \\ &\|JR(\hat{q}_e)\dot{\omega}_r\| \le c_Jc_{dw}\|R(\hat{q}_e)\| \le c_J\bar{c}_{dw} \end{aligned} \tag{17}$$

where $\|\hat{q}_{ev}\| \le 1$, $\|R(\hat{q}_e)\| = 1$, $\|\hat{q}_{e0}I_{3\times3}+S(\hat{q}_{ev})\| = 1$ have been used. Accordingly, it can be concluded that, although $L_{\hat{s}\tau_d}(.)$ contains nonlinear, uncertain, and time-varying terms, there always exists some unknown constants $a_0 \ge 0$, $a_1 \ge 0$, $a_2 \ge 0$, such that

$$\|L_{\hat{s}\tau_d}(.)\| \le a_0 + a_1\|\omega_b\| + a_2\|\omega_b\|^2 = a^T\Phi \tag{18}$$

where $a = [a_0, a_1, a_2]^T$ and $\Phi = [1, \|\omega_b\|, \|\omega_b\|^2]^T$.

It is important to note that Eq. (18) holds for any type of rigid spacecraft. Furthermore, Φ is independent of physical parameters of spacecraft and operation conditions of the vehicle: it represents the core information of the system. Therefore, the following observation can be stated.

Observation 2: For spacecraft satisfying assumptions 1–4, the relation as described in Eq. (18) is always true regardless of external disturbances, uncertain and time-varying inertia matrix, quaternion observation error, or type of spacecraft.

This observation, if used properly, could allow for a truly model-independent attitude control scheme to be developed, as stated in the following proposition.

Proposition 3: Consider a spacecraft with time-varying inertia matrix J, external disturbances, and attitude observer defined in Eqs. (4–6). Design the following quaternion observer-based attitude tracking control scheme:

$$\begin{aligned} \tau(\hat{\omega}_e,\hat{q}_{ev})|_{\tau_d\neq0,\dot{J}\neq0} &= -[k_0+k(t)]\hat{s} \qquad k(t) = \frac{\hat{a}^T\Phi}{\|\hat{s}\|+\varepsilon} \\ \dot{\hat{a}} &= -b_1\hat{a} + b_2\frac{\|\hat{s}\|^2\Phi}{\|\hat{s}\|+\varepsilon}, \qquad \varepsilon = \frac{\mu}{1+\|\Phi\|} \end{aligned} \tag{19}$$

where $\hat{s}(\hat{\omega}_e, \hat{q}_{ev})$ is defined as in Eq. (14), $k_0 > 0$, $\mu > 0$, $b_1 > 0$, and $b_2 > 0$ are control parameters chosen by the designer, $\Phi = [1, \|\omega_b\|, \|\omega_b\|^2]^T$. Then, it is ensured that 1) the estimated attitude and velocity tracking errors approach to their actual values asymptotically, 2) the actual attitude and velocity tracking errors converge to a small set containing the origin, and 3) the closed-loop system is globally stable in that all the internal signal variables are uniformly ultimately bounded and continuous everywhere.

Proof: Consider the following Lyapunov candidate function:

$$V = \frac{1}{2}\hat{s}^TJ\hat{s} + k_0\beta[(1-\hat{q}_{e0})^2 + \hat{q}_{ev}^T\hat{q}_{ev}] + \frac{1}{2b_2}\tilde{a}^T\tilde{a} + \frac{1}{2}\tilde{q}_{ev}^T\tilde{q}_{ev} \tag{20}$$

where $\tilde{a} = a - \hat{a}$. The structural property of J is used in constructing the Lyapunov function to allow for a control scheme to be developed without using J explicitly in the control algorithms. Taking the derivative of V, it follows that

$$\begin{aligned} \dot{V} &= \hat{s}^TJ\dot{\hat{s}} + \frac{1}{2}\hat{s}^T\dot{J}\,\hat{s} + k_0\beta\hat{q}_{ev}^T[\omega_e + \gamma y\times\hat{y}] - \tilde{a}^T\frac{\dot{\hat{a}}}{b_2} - \tilde{q}_{e0}\dot{\tilde{q}}_{e0} \\ &= \hat{s}^T[\tau + L_{\hat{s}\tau_d}(.)] + k_0\beta\hat{q}_{ev}^T\omega_e + k_0\beta\gamma\hat{q}_{ev}^T(y\times\hat{y}) \\ &\quad - \tilde{a}^T\frac{\dot{\hat{a}}}{b_2} - \gamma\tilde{q}_{e0}^2\|\tilde{q}_{ev}\times x\|^2 \end{aligned} \tag{21}$$

Substituting the controller given in Eq. (19) into Eq. (21), we have

$$\begin{aligned} \dot{V} &\le -\frac{k_0}{2}[\|\hat{s}\|^2 + \|\hat{\omega}_e\|^2 + \beta^2\|\hat{q}_{ev}\|^2] - \frac{\|\hat{s}\|^2\hat{a}^T\Phi}{\|\hat{s}\|+\varepsilon} + \|\hat{s}\|a^T\Phi \\ &\quad - \tilde{a}^T\frac{\dot{\hat{a}}}{b_2} + k_0\beta\gamma\hat{q}_{ev}^T(y\times\hat{y}) - \gamma\tilde{q}_{e0}^2\|\tilde{q}_{ev}\times x\|^2 \le -\frac{k_0}{2}[\|\hat{s}\|^2 \\ &\quad + \|\hat{\omega}_e\|^2 + \beta^2\|\hat{q}_{ev}\|^2] + \tilde{a}^T\left(\frac{\|\hat{s}\|^2\Phi}{\|\hat{s}\|+\varepsilon} - \frac{\dot{\hat{a}}}{b_2}\right) + \frac{\varepsilon\|\hat{s}\|a^T\Phi}{\|\hat{s}\|+\varepsilon} \\ &\quad + k_0\beta\gamma\hat{q}_{ev}^T(y\times\hat{y}) \le -\frac{k_0}{2}[\|\hat{s}\|^2 + \|\hat{\omega}_e\|^2 + \beta^2\|\hat{q}_{ev}\|^2] \\ &\quad + \frac{b_1}{b_2}\tilde{a}^T\hat{a} + \mu a_m + k_0\beta\gamma\hat{q}_{ev}^T(y\times\hat{y}) \end{aligned} \tag{22}$$

where $a_m = \max(a_0, a_1, a_2)$. Note that

$$\tilde{a}^T\hat{a} = -\tfrac{1}{2}[\tilde{a}^T\tilde{a} + \hat{a}^T\hat{a}] + \tfrac{1}{2}a^Ta = -(\hat{a}-a/2)^T(\hat{a}-a/2) + \tfrac{1}{4}a^Ta \tag{23}$$

which allows Eq. (22) to be further expressed as

$$\dot{V} \le -\min\left[\frac{k_0}{\lambda_{\max}(J)}, b_1, \frac{\beta}{2}, 1\right]V + c_2 \tag{24}$$

where

$$\begin{aligned} c_2 = \sup_{t\ge0}\Bigg\{&\frac{k_0\beta^2}{2}(1-\hat{q}_{e0})^2 + \frac{b_1}{2b_2}a^Ta + \mu a_m \\ &+ k_0\beta\gamma\|\hat{q}_{ev}^T(y\times\hat{y})\| + \frac{1}{2}\tilde{q}_{ev}^T\tilde{q}_{ev}\Bigg\} < \infty \end{aligned} \tag{25}$$

Thus, the closed-loop system is globally stable. Furthermore, it can be shown from Eqs. (22) and (23) that

$$\dot{V} \le -\frac{k_0}{2}[\|\hat{s}\|^2 + \|\hat{\omega}_e\|^2 + \beta^2\|\hat{q}_{ev}\|^2] + c_3 \tag{26}$$

with

$$c_3 = \sup_{t\ge0}\left\{\frac{b_1}{4b_2}a^Ta + \mu a_m + k_0\beta\gamma\|\hat{q}_{ev}^T(y\times\hat{y})\|\right\} < \infty \tag{27}$$

Clearly, $\dot{V}(t) < 0$ if $\|\hat{\omega}_e(t)\| > \sqrt{2c_3/k_0}$, $\|q_{ev}(t)\| > \sqrt{2c_3/(k_0\beta^2)}$, or $\|\hat{s}(t)\| > \sqrt{2c_3/k_0}$ according to Eq. (26), which implies that $V(t)$ decreases monotonically. The decrease of $V(t)$ eventually drives $\|\hat{s}(t)\|$ and $\|\hat{q}_{ev}(t)\|$ into $\|\hat{s}(t)\| \le \sqrt{2c_3/k_0}$ and $\|q_{ev}(t)\| \le \sqrt{2c_3/(k_0\beta^2)}$ from Eqs. (20) and (26). Moreover, $V(t)$ keeps decreasing unless $\|\hat{\omega}_e\|$ also enters into $\|\hat{\omega}_e(t)\| \le \sqrt{2c_3/k_0}$ from Eq. (26). Therefore, the estimated tracking errors are bounded ultimately as

$$\begin{aligned} \lim_{t\to\infty}[\hat{\omega}_e(t), \hat{q}_{ev}(t)] \in &\left(\|\hat{\omega}_e\| \le \sqrt{\frac{2c_3}{k_0}}, \|\hat{q}_{ev}\| \le \sqrt{\frac{2c_3}{k_0\beta^2}}\right) \\ &\cap \left(\|\hat{\omega}_e + \beta\hat{q}_{ev}\| \le \sqrt{\frac{2c_3}{k_0}}\right) \end{aligned} \tag{28}$$

which is a small set containing the origin $[\hat{\omega}_e, \hat{q}_{ev}] = 0$.

From Lemma 1, we know that $q_o(t) \to q_b(t)$ as $t \to \infty$, thus $\hat{q}_e(t) \to q_e(t)$ and $\hat{\omega}_e \to \omega_e$ as $t \to \infty$. Therefore, it is concluded from Eqs. (27) and (28) that the actual attitude tracking error q_e and angular velocity tracking error ω_e converge to a small set containing the origin, and larger k_0, b_2 and smaller μ, b_1 lead to smaller error set.

Remark 2: The proposed control scheme is motivated by [15]. As the quaternion vector is unavailable for direct feedback, the estimated quaternion variables are incorporated in the algorithm. As such, a new Lyapunov function is employed for stability analysis. Because the algorithm only makes use of the very general (core) information associated with the underlying vehicle, the resultant control scheme turns out to be simpler in structure, less involved in control design, and less demanding for online computation and implementation, as compared with most existing methods.

Remark 3: It is worth noting that the proposed control scheme demands little information about system parameters/dynamics; it is an essentially model-independent attitude tracking control method in that redesign or reprogramming is not needed even if external disturbances occur and system parameters or dynamics change during system operation. To illustrate this point, consider the following possible situations: 1) uncertain dynamics occur due to sudden change or continuous variation of J from J_0 to $J_0 \pm \Delta J$, which leads to $\pm\Delta J\dot{\omega}_b \pm \Delta\dot{J}\omega_b \pm S(\omega_b)\Delta J\omega_b$ into the system dynamics; 2) additional unknown nonlinear term $N(.)$ adds to the model due to, for instance, possible component failures; and 3) unexpected external disturbances act on the system.

Clearly, the preceding situations can be coped with by the proposed control scheme without increasing any difficulty, complexity, or cost in design and implementation, as long as $J(.)$ satisfies assumptions 1–2, $N(.)$ is upper bounded by $\|N\| \le d_0 + d_1\|\omega_b\| + d_2\|\omega_b\|^2$ for some unknown constants $d_0 \ge 0$, $d_1 \ge 0$, and $d_2 \ge 0$, and the disturbances obeys assumption 4. Although there is no need to redesign or reprogram the control scheme to accommodate those uncertainties, one would typically have to perform some degree of control retuning to preserve the performance characteristics if the underlying plant has experienced a significant change.

IV. Robust Adaptive Control Under Actuation Limits

An important consideration in practical application is the torque constraint due to actuation saturation as imposed as

$$|\tau_i| \le \tau_{\max}, \qquad (i = 1, 2, 3) \tag{29}$$

where $\tau = [\tau_1, \tau_2, \tau_3]^T$ and $\tau_{\max} > 0$ denotes the maximum allowable torque of each actuator. For the system under consideration to admit a feasible attitude tracking control scheme, the following assumption is needed:

5) There exists some constant $\tau_m > 0$, such that

$$\tau_{\max} \ge a^T\Phi + \tau_m \tag{30}$$

where τ_m is called the power margin of the control system. This assumption means that the actuation system can supply combined torque sufficient enough to allow the vehicle to perform a given maneuvering operation under external disturbances. A similar assumption has been used in [9,11,13–15] when addressing spacecraft attitude tracking without considering attitude estimation error. To the best of our knowledge, no result is available in the literature that deals with the attitude tracking problem when modeling uncertainties, external disturbances, estimation error, and torque limits occur concurrently.

Proposition 4: Consider the attitude tracking error dynamics given in Eq. (15). Stable attitude tracking as described in Proposition 3 is ensured under assumptions 1–5, if the following control scheme inspired by [15] is implemented:

$$\tau(\hat{\omega}_e, \hat{q}_{ev})|_{\tau_d\neq 0, \dot{J}\neq 0} = -\tau_{\max}\,\mathrm{sat}\left([k_0 + k(t)]\frac{\hat{s}}{\tau_{\max}}\right) \tag{31}$$

with

$$\mathrm{sat}([k_0 + k(t)]\hat{s}/\tau_{\max}) = \begin{cases} \frac{\hat{s}}{\|\hat{s}\|}, & \text{if} \|\hat{s}\| > \tau_{\max}/k_0 + k(t) \\ [k_0 + k(t)]\hat{s}/\tau_{\max} & \text{otherwise} \end{cases} \tag{32}$$

where $k(t)$ and other variables and parameters are defined as in Eq. (19).

Proof: The stability of the control scheme is verified by examining two cases: case 1) $\|\hat{s}\| \le \tau_{\max}/[k_0 + k(t)]$ and case 2) $\|\hat{s}\| > \tau_{\max}/[k_0 + k(t)]$.

Case 1: $\|\hat{s}\| \le \tau_{\max}/[k_0 + k(t)]$. Under this case, the controller (31) becomes $\tau = -[k_0 + k(t)]\hat{s}$. Considering the Lyapunov candidate function as in Eq. (20), it follows that

$$\begin{aligned}\dot{V} &= \hat{s}^T[\tau + L(.)] + k_0\beta\hat{q}_{ev}^T\omega_e + k_0\beta\gamma\hat{q}_{ev}^T(y \times \hat{y}) \\ &- \tilde{a}^T\frac{\dot{\hat{a}}}{b_2} - \gamma\tilde{q}_{e0}^2\|\tilde{q}_{ev} \times x\|^2 \le -\frac{k_0}{2}[\|\hat{s}\|^2 + \|\hat{\omega}_e\|^2 \\ &+ \beta^2\|\hat{q}_{ev}\|^2] + \frac{b_1}{b_2}\tilde{a}^T\hat{a} + \mu a_m + k_0\beta\gamma\hat{q}_{ev}^T(y \times \hat{y})\end{aligned} \tag{33}$$

The result thus is established using the same argument as in the proof of Proposition 3.

Case 2: $\|\hat{s}\| > \tau_{\max}/[k_0 + k(t)]$, $\tau = -\tau_{\max}\hat{s}/\|\hat{s}\|$, which satisfies $|\tau_i| \le \tau_{\max}$, $(i = 1, 2, 3)$. We need to address two subcases:

A) Suppose $\tilde{q}_{e0}(0) = q_b^T(0)q_o(0) \ne 0$, the Lyapunov candidate function is chosen as

$$V = \frac{1}{2}\hat{s}^T J\hat{s} + 2\beta[(1 - \hat{q}_{e0})^2 + \hat{q}_{ev}^T\hat{q}_{ev}] + \frac{16\gamma}{\tilde{q}_{e0}^2}\tilde{q}_{ev}^T\tilde{q}_{ev} \tag{34}$$

and its time derivative can be determined as

$$\begin{aligned}\dot{V} &= \hat{s}^T J\dot{\hat{s}} + \frac{1}{2}\hat{s}^T\dot{J}\,\hat{s} + 2\beta\hat{q}_{ev}^T\hat{\omega}_e + 2\beta\gamma\hat{q}_{ev}^T(y \times \hat{y}) \\ &- \frac{32\gamma}{\tilde{q}_{e0}^3}\dot{\tilde{q}}_{e0} = \hat{s}^T[\tau + L^*(.) - \hat{s}] + 2\beta\hat{q}_{ev}^T\hat{\omega}_e + 2\beta\gamma\hat{q}_{ev}^T(y \times \hat{y}) \\ &- \frac{32\gamma^2}{\tilde{q}_{e0}^2}\|\tilde{q}_{ev} \times x\|^2(\tilde{q}_{e0}^2 + \tilde{q}_{ev}^T\tilde{q}_{ev}) \le -\frac{\tau_{\max}}{\|s\|}\hat{s}^T\hat{s} + \|\hat{s}\|a^T\Phi \\ &- \|\hat{\omega}_e\|^2 - \beta^2\|\hat{q}_{ev}\|^2 + 2\beta\gamma\|\hat{q}_{ev}\|\|(y \times \hat{y})\| - 32\gamma^2\|\tilde{q}_{ev} \times x\|^2 \\ &\le -\|\hat{s}\|\tau_m - \|\hat{\omega}_e\|^2 - \beta^2\|\hat{q}_{ev}\|^2 + 8\beta\gamma\|\hat{q}_{ev}\|\|\tilde{q}_{ev} \times x\| \\ &- 32\gamma^2\|\tilde{q}_{ev} \times x\|^2 \le -\tau_m\|\hat{s}\| - \|\hat{\omega}_e\|^2 - \frac{\beta^2}{2}\|\hat{q}_{ev}\|^2 < 0\end{aligned} \tag{35}$$

where $L^*(.) = L_{\hat{s}\tau_d}(.) + \hat{s}$, which obviously still satisfies $\|L^*\| \le a^T\Phi$. From Eq. (35), it is deduced that $\hat{\omega}_e, \hat{q}_{ev} \in \mathcal{L}_2 \cap \mathcal{L}_\infty$, and then $\dot{\hat{q}}_{ev}, \omega_b, \Phi, \hat{a}, \dot{\hat{a}}, k \in \mathcal{L}_\infty$, and $\dot{\hat{\omega}}_e \in \mathcal{L}_\infty$. Therefore, $[\hat{\omega}_e, \hat{q}_{ev}(t)]$ will be eventually driven to case 1, otherwise using Barbalat's lemma, $\lim_{t\to\infty}[\hat{\omega}_e, \hat{q}_{ev}(t)] = 0$ based on the fact $\hat{\omega}_e, \hat{q}_{ev} \in \mathcal{L}_2 \cap \mathcal{L}_\infty$, and $\dot{\hat{\omega}}_e, \dot{\hat{q}}_{ev} \in \mathcal{L}_\infty$.

B) Suppose $\tilde{q}_{e0}(0) = q_b^T(0)q_o(0) = 0$. For this subcase, the Lyapunov candidate function defined Eq. (34) is invalid due to $\tilde{q}_{e0}(0) = 0$, so that a new Lyapunov candidate function is needed to analyze the stability. Let us choose it as follows:

$$V = \tfrac{1}{2}\hat{s}^T J\hat{s} + 2\beta[(1 - \hat{q}_{e0})^2 + \hat{q}_{ev}^T\hat{q}_{ev}] + \tfrac{1}{2}\tilde{q}_{ev}^T\tilde{q}_{ev} \tag{36}$$

Upon taking the time derivative of V with respect to time, we have

$$\begin{aligned}\dot{V} &= \hat{s}^T J\dot{\hat{s}} + \frac{1}{2}\hat{s}^T\dot{J}\,\hat{s} + 2\beta\hat{q}_{ev}^T\hat{\omega}_e + 2\beta\gamma\hat{q}_{ev}^T(y \times \hat{y}) - \tilde{q}_{e0}\dot{\tilde{q}}_{e0} \\ &= \hat{s}^T\tau + \hat{s}^T[L^*(.) - \hat{s}] + 2\beta\hat{q}_{ev}^T\hat{\omega}_e + 2\beta\gamma\hat{q}_{ev}^T(y \times \hat{y}) \\ &\le -\frac{\tau_{\max}}{\|s\|}\hat{s}^T\hat{s} + \|\hat{s}\|a^T\Phi - \|\omega_e\|^2 - \beta^2\|\hat{q}_{ev}\|^2 + 2\beta\gamma\|\hat{q}_{ev}\| \\ &\le -\|\hat{s}\|\tau_m - \|\omega_e\|^2 - (\beta\|\hat{q}_{ev}\| - \gamma)^2 + \gamma^2 \\ &= -\|\hat{s}\|\tau_m - \|\omega_e\|^2 - \beta\|\hat{q}_{ev}\|(\beta\|\hat{q}_{ev}\| - 2\gamma)\end{aligned} \tag{37}$$

Then, it can be concluded by using similar analysis to that used in Eq. (26) that

$$\begin{aligned}\lim_{t\to\infty}[\hat{\omega}_e(t), \hat{q}_{ev}(t)] &\in \left(\|\omega_e\| \le \gamma, \|\hat{q}_{ev}\| \le \frac{2\gamma}{\beta}\right) \\ &\cap \left(\|\hat{\omega}_e + \beta\hat{q}_{ev}\| \le \frac{\gamma^2}{\tau_m}\right)\end{aligned} \tag{38}$$

That is, $\hat{\omega}_e, \hat{q}_{ev} \in \mathcal{L}_\infty$, which implies that $\dot{\hat{q}}_{ev}, \omega_b \in \mathcal{L}_\infty$ and $\Phi, \hat{a}, \dot{\hat{a}}, k \in \mathcal{L}_\infty$. Thus, ultimately uniformly bounded stability is achieved for this subcase.

To summarize, for all the possible cases, the preceding analysis indicates that the control scheme given in Eq. (31) ensures stable attitude tracking in the presence of external disturbances, uncertain or even time-varying system parameters, and attitude estimate error, as well as torque limit.

Remark 4: It is worth mentioning that our attitude control scheme is derived under the following extreme conditions: 1) little information on the vehicle about its nonlinear dynamics, modeling uncertainties, and physical parameters; 2) limited information on its operational environment in terms of operational conditions and external disturbances; 3) torque limits; and 4) attitude estimation error. As such, the developed control scheme is more practical and feasible as compared with most existing strategies.

V. Simulation Study

We now conduct numerical simulation studies to validate the effectiveness of the quaternion observer-based robust adaptive controller developed in the previous section. The same inertia matrix and initial conditions as used by [16] are used in the simulation:

$$J = J_0 = \begin{bmatrix} 20 & 1.2 & 0.9 \\ 1.2 & 17 & 1.4 \\ 0.9 & 1.4 & 15 \end{bmatrix}$$

$$q_b(0) = [\sqrt{8}/3, 1/\sqrt{27}, 1/\sqrt{27}, 1/\sqrt{27}]^T$$

$$q_r(0) = [1, 0, 0, 0]^T, \qquad \omega_b(0) = [0, 0, 0]^T$$

$$q_o(0) = [\sqrt{3}/2, 1/\sqrt{12}, 1/\sqrt{12}, 1/\sqrt{12}]^T \quad \text{for } q_b^T(0)q_o(0) \neq 0$$

$$q_o(0) = [-1/3, 2\sqrt{6}/9, 2\sqrt{6}/9, 2\sqrt{6}/9]^T \quad \text{for } q_b^T(0)q_o(0) = 0$$

The reference angular velocity is given as $\omega_r(t) = z(t)[1, 1, -1]^T$, where $z(t)$ is defined as

$$z(t) = 0.3\cos t(1 - e^{-0.01t^2}) + (0.08\pi + 0.006\sin t)te^{-0.01t^2}$$

Any time-varying unit vector with bounded time derivative can be chosen as the reference signal for the observer. In our simulation, we select the same $x(t) = [\cos t, \sin t, 0]^T$ as used in [16].

The simulation is to explicitly address external disturbances, unknown time-varying systems parameters, torque limits, and unavailability of direct/accurate measurement of quaternion attitude, as usually encountered in practice. Virtually all existing model-dependent attitude control methods are inapplicable directly in this case. To test the effectiveness of the proposed control schemes defined in Eqs. (19) and (31) in dealing with unknown and time-varying $J(.)$ as well as $\tau_d(.)$, the following disturbance torque

$$\tau_d = 5(\|\omega\|^2 + 0.3)[\cos 0.2t, \sin 0.5t, \cos 0.8t]^T$$

and time-varying moment inertia matrix

$$J = J_0 + \Delta J$$

$$\Delta J = \text{diag}([3, 2, 1])[1 + e^{-0.1t} + 2u(t - 10) - 4u(t - 20)]$$

are incorporated into the model, where J_0 is the constant portion of $J(t)$, and $u(.)$ is defined as $u(t \geq 0) = 1$ and $u(t < 0) = 0$. Note that both continuously time-varying and jump change of the diagonal element of $J(.)$ is involved, as shown in Fig. 1, arisen from, for instance, fuel consumption (burning) and/or payload release/addition.

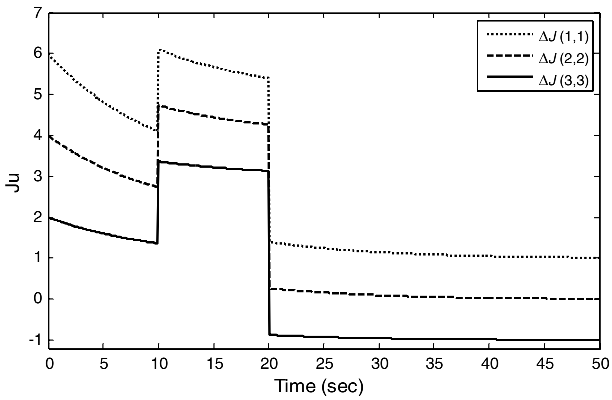

Fig. 1 Continuously time-varying and jump variation of the diagonal element of the inertia matrix (due to, for instance, fuel consumption and/or payload release/addition).

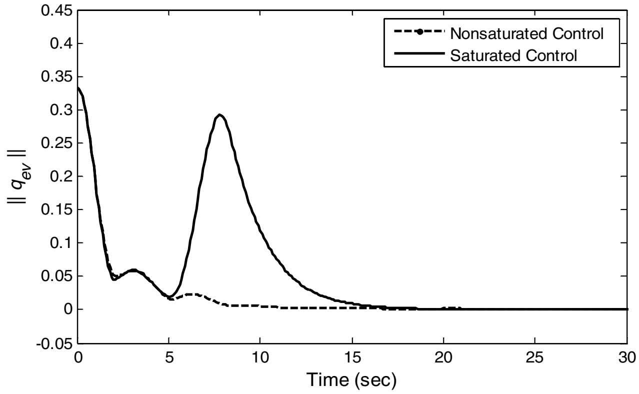

Fig. 2 Quaternion attitude tracking error norm under $q_b^T(0)q_o(0) \neq 0$ (with/without torque limits).

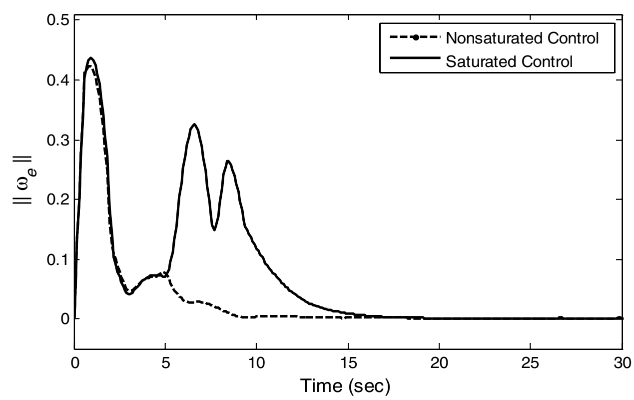

Fig. 3 Angular velocity tracking error norm under $q_b^T(0)q_o(0) \neq 0$ (with/without torque limits).

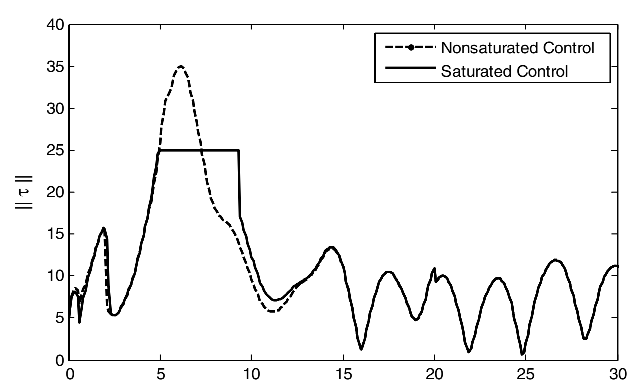

Fig. 4 Control torque norm under $q_b^T(0)q_o(0) \neq 0$ (with/without torque limit).

To achieve high precision control with smooth control action, the control parameters are chosen as

$$\beta = 1, \qquad k_0 = 10, \qquad b_1 = 0.002, \qquad b_2 = 30$$

for both controllers as defined in Eqs. (19) and (31). The parameter μ is a control parameter that can be chosen freely with a wide range: $\mu = 0.002$ for the controller in Eq. (19) and $\mu = 0.04$ for the saturated controller in Eq. (31) with $\tau_{\max} = 25$. Simulation results are presented in Figs. 2–7. It is interesting to note that the proposed control schemes lead to fairly good control performance without using any information on $\tau_d(.)$ or $J(.)$ (not even J_0) for both $q_b^T(0)q_o(0) \neq 0$ and $q_b^T(0)q_o(0) = 0$. In setting up and implementing the control scheme, one only needs to select the nonrestrictive control parameters β, k_0, b_1, b_2, and μ. Analysis and simulation show that larger β, k_0, b_2, and smaller b_1, μ lead to better control performance (quicker response and smaller steady error) at the price of a larger starting (initial) control signal, thus certain tradeoffs might be necessary in practice.

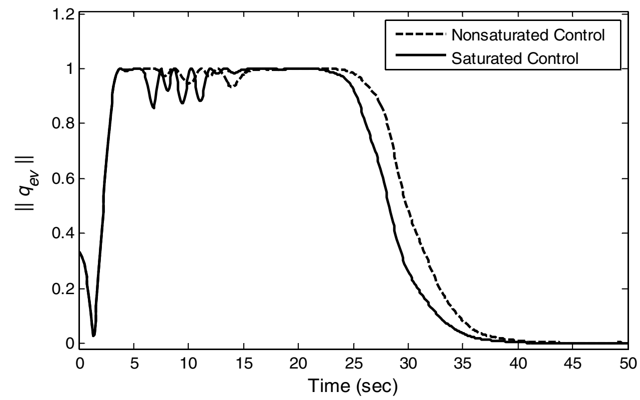

Fig. 5 Quaternion attitude tracking error norm under $q_b^T(0)q_o(0) = 0$ (with/without torque limits).

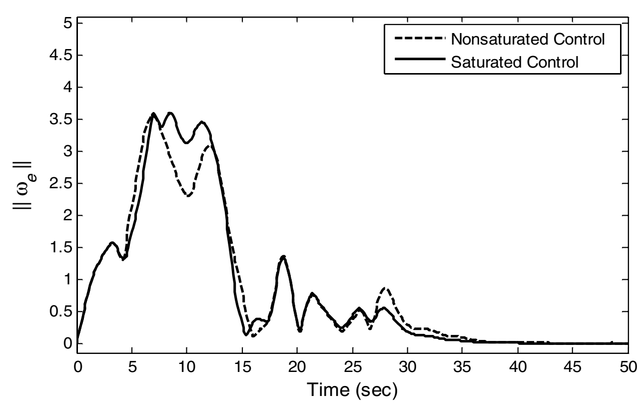

Fig. 6 Angular velocity tracking error norm under $q_b^T(0)q_o(0) = 0$ (with/without torque limits).

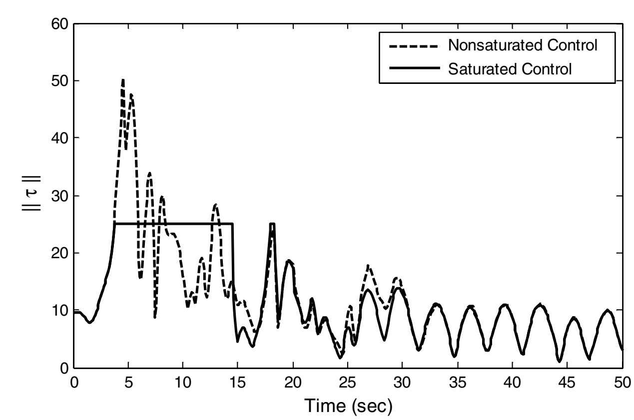

Fig. 7 Control torque norm under $q_b^T(0)q_o(0) = 0$ (with/without torque limits).

VI. Conclusions

Attitude tracking control of spacecraft under extreme operation conditions is studied in this work. Under mild assumptions that always hold for rigid-body spacecraft, a new attitude tracking control scheme without using direct and accurate attitude measurement is developed. The novelty of the proposed control strategy also lies in its robustness against inevitable external disturbances and its independence of system parameters/dynamics; the dynamic model of the rigid-body spacecraft only plays a role in stability analysis but is never needed for control setup and implementation. Furthermore, the control scheme can be setup effortlessly by select certain nonrestrictive control parameters. These features make it possible to continuously maintain high-precision attitude tracking control of the vehicle during its entire operation, without the need for completely redesigning or reprogramming of the controller under widely varying operation conditions.

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